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MECHANICAL FORCE ON DIELECTRIC POLARIZATION CURRENT IN A MAGNETIC FIELD

by

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ABSTRACT

In the early formulation of electromagnetic theory, a mechanical force on a dieletric in a time-varying field was hypothesized in the form of a Poynting energy flow term. Based on the molecular view of the polarization current in both polar and nonpolar dielectrics, a part of this force can be shown to be just the dielectric counterpart of the familiar **J x B** force in conductors. Experimental detection of this dielectric force is described. From a conceptual standpoint, this places conductors and dielectrics on a symmetric basis in the context of electromechanical force. From an applicability standpoint, it may find possible uses in micromotion devices and other areas of emerging technology.

I. INTRODUCTION

There appears to be no direct answer in the standard literature (see e.g. Refs. 1 and 2) to the question whether or not a polarization current in a neutral dielectric in a magnetic field experiences a mechanical force. Early discussion converging on this subject[3,4] developed into various hypotheses related to the distinction between mechanical and electromagnetic momenta in electromagnetic field in ponderable matter. Nor do there seem to be any conclusive experiments bearing on this question. If the mechanical force described above does exist, then it would provide the dielectric counterpart of the similar force on a conductor, and would thus extend the existing symmetry in the electromagnetic theory between conductors and dielectrics. Furthermore, once the existence of this force is demonstrated, its possible practical applications can be explored in the context of present-day technology. These considerations provide the impetus for taking up anew the discussion of what had been relegated to an obscure nuance of electromagnetic theory.

While the force under discussion has been invoked earlier as a basis for development of the dielectric analog of the science of magnetohydrodynamics,[5] no theoretical justification or experimental evidence has been available.

II. BACKGROUND

For a treatment of the problem in terms of electromagnetic the fields **E**, **D**, **H** and **B**, (the electric field, the displacement vector, the magnetic field and the magnetic induction,

respectively), we consider the Maxwell stress tensor in *time-varying* field. A concise discussion in this context has been presented by Stratton[1] (Cf. Section 2.29 of his book). Consider for this discussion a neutral volume (i.e. one with no net charges) of a non-magnetic (i.e. the relative magnetic permeability $\mu = 1$) and non-conducting medium with a relative dielectric constant ϵ that is constant in space. Then, in a time-varying field, the rate of change of the net momentum **G** (i.e., the sum of the mechanical momentum $\mathbf{G}_{mech}$ and the electromagnetic momentum $\mathbf{G}_{em}$) in a volume V of the medium is found to be

$$\dot{\mathbf{G}} = \int_V \frac{\partial}{\partial t} (\mathbf{D} \times \mathbf{B}) \; dV \tag{1}$$

The mechanical force **F** on the volume is $\mathbf{F} = \dot{\mathbf{G}}_{mech}$. The discussion now proceeds as follows: If the right hand side of Eq. (1) could be clearly separated into what is mechanical and what is electromagnetic, then the force **F** could be identified. How to do this is not at all obvious, and various hypotheses have been suggested. According to one hypothesis, one subtracts out the electromagnetic momentum, taken to be $\mathbf{g}_{em} = \mu_o \epsilon_o \dot{\mathbf{S}}$ per unit volume, from the right hand side of Eq. (1) (μ_o , ϵ_o = the magnetic permeability and the permittivity of free space, respectively; $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ is the Poyinting vector), and what is left is the mechanical momentum. This leads to the expression for the force

$$\mathbf{F} = \int_V \mu_o \epsilon_o \chi \dot{\mathbf{S}} \; dV \tag{2}$$

where $\chi = \epsilon - 1$ is the polarizability of the medium. We can

rewrite the above equation as a density force

$$\mathbf{F} = \chi \epsilon_o \dot{\mathbf{E}} \times \mathbf{B} + \chi \epsilon_o \mathbf{E} \times \dot{\mathbf{B}} \qquad (3)$$

The first term on the right hand side is the force $\mathbf{J}_p \times \mathbf{B}$ (since by definition $\mathbf{J}_p = \chi \epsilon_o \dot{\mathbf{E}}$, the polarization current density), analogous to the force $\mathbf{J} \times \mathbf{B}$ in conductors. This, however, leaves the last term with no clear interpretation as a material force. Thus, while we are able to extract out a force term $\mathbf{J}_p \times \mathbf{B}$, it only comes about as a hypothesis and only in conjunction with a force term whose physical origin is unclear.

Microscopic treatment of electric current, including molecular contributions that contain the polarization current, has been discussed in the early literature.[2] There appears to be no suggestion, however, that one could obtain a net mechanical force by taking the cross product of each molecular contribution to the current with an externally impressed magnetic field. Indeed, such a suggestion would not stand by itself without justification such as discussed in the next section.

The force $\mathbf{F}$ in Eq. (3) has sometimes been referred to as the 'Abraham density force' (for the case $\mu = 1$), and has been rooted in a long-standing debate between the forms of the electromagnetic energy-momentum tensor proposed by Einstein and Laub, and Abraham on one hand, and Minkowski on the other[3,4]. Experimental observation of this force has traditionally been considered difficult because of its smallness.[6] One such experiment uses a ferromagnetic material,[7] and is thus inconclusive with regard to the pure dielectric effect. A more

relevant set of experiments uses a non-magnetic dielectric material, ceramic Barium Titanate.[8] However, this material is also strongly piezoelectric. The interpretation of the experiment did not include a discussion of this effect, although the quantitative agreement of the results with the prediction from the first term of Eq. (3) may be evidence that the piezoelectric forces were unimportant.[9] The main difficulty with all the experiments to date has been to clearly isolate and demonstrate the presence of a mechanical force on a dielectric polarization current in a magnetic field.[10]

III. MICROSCOPIC DERIVATION

We now attempt to derive the force on a magnetized dielectric carrying a polarization current from first principles by referring to the particulate electric current. The point of this elementary discussion is that if a mechanical force described by the last term of Eq. (3) does exist, it must be understandable from a microscopic point of view.[11] We start with the Lorentz force **f** on a single charge of magnitude q moving with a velocity **v** in a magnetic field:[2]

$$\mathbf{f} = q\,(\mathbf{v} \times \mathbf{B}) \tag{4}$$

If there is also an ambient electric field **E** present, then there is an additional force $q\,\mathbf{E}$ on the charge. In the present instance, this force gives rise to the electric current which will be treated separately.

Dielectric materials, for the present discussion, may be broadly divided into two classes: polar and nonpolar[12] (Fig. 1). In a polar dielectric the molecules are permanent electric dipoles and are randomly oriented in absence of an electric field. When a time-varying electric field is applied, the dipoles continually align themselves with the field, and the consequent displacement of the charges results in the polarization current. We may imagine here that the dipoles rotate about fixed centroids. In nonpolar dielectrics there is no charge separation in the atoms/molecules in the absence of an electric field. In the field, the atoms or molecules become polarized, i.e., there arise a separation of positive and negative charges, and a consequent polarization current. We may imagine that the positive and the negative charges in an atom or molecule move towards or away from a centroid. In the following we will discuss simultaneously both these classes of dielectrics to show that the discussion is universal. Since electric current is simply the rate of flow of charges, it follows, referring to Fig. 1, that the polarization current density is

$$\mathbf{J}_p = \Sigma\ (q^+ \mathbf{v}_{\parallel}^+ + q^- \mathbf{v}_{\parallel}^-) + \Sigma\ (q^+ \mathbf{v}_{\perp}^+ + q^- \mathbf{v}_{\perp}^-) \qquad (5)$$

where q^+ and q^- are the positive and the negative charges inside an atom or a molecule, and $\mathbf{v}_{\parallel}$ and $\mathbf{v}_{\perp}$ are their velocity components parallel and perpendicular to the applied electric field. The summations are performed over all the molecules in a unit volume. The second summation arises only in the case of polar dielectrics, but vanishes statistically since as many molecules rotate in one sense as another. Hence

$$\mathbf{J}_p = \Sigma \left(q^+ \mathbf{v}_{\parallel}^+ + q^- \mathbf{v}_{\parallel}^- \right) \tag{6}$$

By the Lorentz force law (Eq. (4)) there is a microscopic force $\mathbf{f}_{\perp}^+ = q^+ (\mathbf{v}_{\parallel}^+ \times \mathbf{B})$ on each positive charge and $\mathbf{f}_{\perp}^- = q^- (\mathbf{v}_{\parallel}^- \times \mathbf{B})$ on each negative charge arising from the velocity components parallel to the electric field. It will be seen that these two forces are in the same sense, i.e., they add up to a net force on the molecule. Similarly, the velocity components perpendicular to the field (in polar dielectrics) give rise to forces $\mathbf{f}_{\parallel}^+$ and $\mathbf{f}_{\parallel}^-$ that add. Thus there arises a volume force on *any* dielectric

$$\begin{aligned} \mathbf{F} &= \Sigma \left(\mathbf{f}_{\perp}^+ + \mathbf{f}_{\perp}^- \right) + \Sigma \left(\mathbf{f}_{\parallel}^+ + \mathbf{f}_{\parallel}^- \right) \\ &= \Sigma \left(q^+ \mathbf{v}_{\parallel}^+ + q^- \mathbf{v}_{\parallel}^- \right) \times \mathbf{B} + \Sigma \left(q^+ \mathbf{v}_{\perp}^+ + q^- \mathbf{v}_{\perp}^- \right) \times \mathbf{B} \\ &= \mathbf{J}_p \times \mathbf{B} \end{aligned} \tag{7}$$

since the second summation vanishes statistically for the reason stated above (as many molecules have the parallel forces pointing to the right as to the left). Thus a dielectric carrying a polarization current in a magnetic field is found to be subject to formally the same force as a conductor. The assumption of statisticity in the above derivation should be a good one in solids and liquids, but perhaps questionable in the case of gases.

Since by definition $\mathbf{J}_p = \chi \epsilon_o \dot{\mathbf{E}}$, the above equation reduces to

$$\mathbf{F} = \chi \epsilon_o \dot{\mathbf{E}} \times \mathbf{B} \tag{8}$$

Thus, the above discussion of the molecular view of arising of the $\mathbf{J}_p \times \mathbf{B}$ force does not shed any light on the last term of Eq. (3). If B varies with time, it will contribute an induced

component to the electric field **E**, and Eq. (8) will follow again - with **E** and **B** written as appropriate functions of space and time. There does not emerge a $\chi \epsilon_o \mathbf{E} \times \dot{\mathbf{B}}$ mechanical force term from the above microscopic consideration.[13] In any event, in the experiment described below $\dot{\mathbf{B}} = 0$, so that the remainder of our discussion is independent of the nature of that term.

IV. EXPERIMENTAL CONSIDERATIONS

For a sinusoidally varying electric field of amplitude E_o and frequency ν:

$$E = E_o \sin (2 \pi \nu t) \tag{9}$$

and a static magnetic field B_o perpendicular to the electric field, the force F may be written as

$$F = 2 \pi \nu \chi \epsilon_o E_o B_o \cos (2 \pi \nu t) \tag{10}$$

We can examine this equation with a view to designing a feasible experiment that maximizes the probability of detecting the force. Use of very high frequencies (e.g., tens of MHz) to maximize the force is not practical, since there are no reliable techniques of measuring mechanical forces at such frequencies. Also, the higher the frequency, the more difficult it is to produce a strong electric field E_o . Very high magnetic fields (e.g., a few Teslas) in the laboratory are invariably such that all the sensors for measuring the force will also have to be placed in this magnetic field, thereby complicating the interpretation of the experiment. These considerations leave us with the

alternative of using a material of as high a dielectric constant as possible. Such a material should have as low a loss tangent as possible, since the presence of a lossy component of the current would complicate interpretation of the experiment. With these considerations in mind, an experiment was designed to detect and identify the force, and to provide an approximate quantitative verification of Eq. (8).

The dielectrics chosen for the experiment are high dielectric constant ceramic materials Lead Zirconate Titanate ("LZT") and Barium Titanate ("BT"), with relative dielectric constants at the frequency of measurement of about 2600[14] and 1700[15] respectively, in the direction of the electric field to be applied (which is the direction of the maximum dielectric constant). At the frequencies of the measurements, these dielectrics are essentially lossless. The dielectric sample is in the shape of a rectangular slab (Fig. 2) with dimensions x = 0.0127 m, y = 0.0257 m, and d = 0.0027 m. The copper capacitor plates (0.0120 m x 0.0250 m) are used to apply the alternating electric field E (corresponding to a sinusoidal voltage of 260 V rms, or a peak voltage V_m of 366 V). The amplitude of this field is $E_o = V_m/d$ = 136 KV/m. The dielectric force is now expected to cause the sample slab to displaced up and down through microscopic distances (of the order of tens of Ångstroms) in the direction perpendicular to both **E** and B_o. The slab is placed between two capacitor plates which are mechanically secured to a rigid (nonconducting and nonmagnetic) table so that they are not free to vibrate. The ambient magnetic field B_o (= 0.6 T, derived from a permanent horseshoe magnet) is applied perpendicular to

E. A high sensitivity quartz accelerometer (PCB Piezoelectric Model 308B02, with a frequency range of 10 Hz to 3 KHz, a mounted resonant frequency of 33.5 KHz, a calibration constant of 1016 mV/ g and a resolution of 0.0005 g in normal operation, g being the acceleration due to gravity) is attached rigidly to and positioned above the slab (outside the pole region of the magnet) to detect vibrations of the slab. The accelerometer generates a voltage proportional to the acceleration, being positive when the acceleration vector is directed upwards. It has been designed to be mounted on large vibrating surfaces and calibrated for that application. At high frequencies and depending on the mounting arrangement the response may be enhanced, but is impossible to predict. We will discuss below the use of this device for our special application.

One property of the high dielectric constant ceramics is that they are also piezoelectric so that they expand and contract when a sinusoidal electric field is applied across them, whether or not a magnetic field is present.[16] Thus when the electric field is applied parallel to the d-dimension, the y-dimension will expand and contract. A small fraction of this energy couples into the accelerometer. The expansion is maximum when the field is maximum in one direction, and the contraction is maximum when the the field is maximum in the opposite direction. The resulting accelerometer voltage can have a complicated form depending on how the sample slab and the accelerometer are mounted. However, if we record the accelerometer voltages both when the sample is in the magnetic field and when it is outside the field, any

difference between these two measurements can be taken to be the effect we are seeking to detect, regardless of how complex the piezoelectric signal is. A magnetic field of the strength employed is not known to alter the piezoelectric response itself. The piezoelectric resonance frequencies of the samples used are far higher than the relevant frequencies of the experiment.[16]

In practice the presence of the piezoelectric signal becomes an advantage for the experiment, by providing a phase reference for the capacitor voltage in the accelerometer output. Since the expansion and contraction of the sample slab are in phase with the electric field, the acceleration (which is 180 degrees out of phase with displacement) is 180 degrees out of phase with his field. Hence the piezoelectric signal is 180 degrees out of phase with respect to the applied voltage.

As mentioned earlier, the accelerometer has been designed to detect vibration of large surfaces. When mounted on a small, light and unsecured wafer, and used at very low acceleration levels, the high frequency resonance of the accelerometer (in this case near 60 KHz, about twice the resonance frequency in normal application) dominates the piezoelectric response (at the frequency ν) in or outside the magnetic field. Furthermore, this response shows a strong instability towards breaking up into a signal at a frequency 2ν . Therefore a damping weight on a rubber pad is placed on the accelerometer. This provides a static stress, seating the sample slab heavily between the capacitor plates. The result of this is to suppress the high frequency amplitudes, to stabilize the low frequency signal

against frequency-doubling and to raise the positive peaks of this signal. All of these factors increase the probability of detection of the signal being searched for. However, because of the above modifications, the accelerometer calibration in normal operation is no longer be valid. The output voltage of the accelerometer is recorded by means of a high sensitivity (1 mV per vertical division) digitizing oscilloscope (Hewlett Packard Model 54120B).

The magnitude of the expected sinusoidal force on the dielectric slab is the product of force $J_p B_o = \chi \epsilon_o \dot{E} B$ per unit volume, and the volume xyd of the slab. Since the acceleration a is proportional to the force, we can write

$$a = 2 \pi \nu K x y d \chi \epsilon_o E_o B_o \cos (2 \pi \nu t) \qquad (11)$$

where the constant K is the inverse of the *effective* mass being accelerated. It is a function of the sample mass (= 0.0063 kg for LZT and 0.0044 kg for BT), the mass of the accelerometer (= 0.073 kg) and possibly also the mass of the damping weight (= 1.1 kg). However, since the high frequencey vibrations cannot be transmitted through the rubber pad, the effective mass is likely to be the sum of the masses of the sample and the accelerometer. Since the accelerometer is much heavier, we may assume that K is largely independent of the sample mass.

We now note from Eq. (11) that the acceleration to be detected is (i) 90 degrees out-of-phase with the applied electric field; (ii) for a given material, proportional to the frequency ν (assuming that K is independent of the frequency); and (iii) for different

materials, proportional to the polarizabilty χ or the relative dielectric constant ϵ . The simultaneous presence of these attributes constitutes a rigorous test of there being a $\mathbf{J}_p$ **x** **B** force on the dielectric. If the vibrations were somehow induced by the conventional **J x B** force on the metallic components of the experiment, then none of the above conditions would hold.

V. RESULTS AND DISCUSSION

The upper frequency limit of the experiment is 3 KHz, set by the accelerometer. Below about 2.5 KHz, the signal being searched for becomes buried in high frequency signal. Two sets of data are presented: LZT at 3 KHz, and BT at 2.5 KHz.

Figure 3a shows the detailed measurement data for the experiment with LZT at 3 KHz. The top panel shows a sampled portion V_{cs} of the capacitor voltage as a function of time. The next two panels show the accelerometer voltages V_o and V_B when the slab is outside the magnetic field and when it is in the field, respectively. The last panel is the difference V_p ($= V_B - V_o$) between the above two signals, and is attributed to the $\mathbf{J}_p$ **xB** force on the dielectric. Figure 3b shows the last panel only for BT at 2.5 KHz.

Figure 4 shows the Fourier-trasformed 'power' (amplitude-squared) spectra corresponding to the last three panels of Fig. 3a. Here, the low and the high frequency components of the observed signal are evident. Also evident is the change in the low frequency signal upon applying the magnetic field. We discard the high frequency component and invert the remaining data to obtain the

filtered waveforms shown in dotted lines in the last three panels of Fig. 3a.

The lack of symmetry of the accelerometer voltages about the time axis, and their distortion from the sinusoidal form are due to the asymmetric mounting of the slab: it is seated at the bottom, and supports a weight at the top. The time-average value of V_p is found to be zero for both LZT and BT. A developing tendency towards frequency-doubling may be noted in V_p of Fig. 3a, again owing to the particular mounting of the slab and the accelerometer.

First we consider the question of the phase of V_p. It was noted earlier that the piezoelectric voltage V_o is 180 degrees out of phase with the applied voltage. Thus, if V_p indeed has a phase difference of 90 degrees with respect to the applied voltage, it should also have the same difference with respect to V_o. We now note from Fig. 3a that the positive peaks of V_p coincide approximately with the zero-crossings of V_o, indicating a phase difference of about 90 degrees. This is also found to be the case with Barium Titanate.

The second test concerns the fact that the amplitude V_{po} of the voltage V_p should be proportional to the frequency ν for the same dielectric material. Because of the skewness of V_p and the presence of the high frequency component, we cannot obtain the amplitude V_{po} of an undistorted sine wave. We therefore test the above proportionality as follows. We take the peak positive value V_{fo} of the filtered voltage as being *proportional* to V_{po}.

This peak value corresponds to the maximum upward-directed acceleration vector acting on the slab.

The V_{fo} found in this way for LZT is 5.0 mV at 2.5 KHz and 6.2 mV at 3 KHz. These voltages are found to be approximately in the ratio of the two frequencies. Lastly, the V_{fo} for BT at 2.5 KHz is found to be 3.6 mV. This is in the ratio of 0.72 with the value 5.0 mV for LZT at 2.5 KHz. This ratio is comparable to the ratio of the dielectric constants, 1700/2600 = 0.65.

The above experiment, while detecting and identifying the force under discussion, does not permit a direct quantitative verification of Eq. (8) (or equivalently, Eq. (11)) because of a lack of knowledge of the modified accelerometer calibration constant. However, if we calculate the accelerations from Eq. (11) and from the measurements (upon estimation of an equivalent sine wave) using the unmodified calibration constant, the latter is larger by a factor $\lesssim 10^2$. Thus, if the accelerometer response has been improved by at least factor of 10, which is not unreasonable, then the two accelerations are within an order-of-magnitude. Considering the various uncertainties of the experiment, this is all that can be said.

Finally, a null experiment was performed where the entire experimental procedure was repeated with a non-conducting, non-piezoelectric, low dielectric constant material (Duroid, $\epsilon \approx 10$). No signal was recorded in or outside the magnetic field.

VII. REMARKS

One may speculate on the practical applicability of the force discussed here. The latter has found wide applications in micromotion devices, e.g. in Scanning Tunneling Microscopy (STM)[17] and atomic scale manipulations using the STM tip.[18] The difference between the two forces is of course that the dielectric force arises only in time-varying fields. Our experiment shows that the even in modest magnetic and electric fields the dielectric force is able to actuate a mechanical device (the accelerometer). It also shows that Eq. (8) may be used as an order-of-magnitude estimate of the acceleration or the displacement. We further note that it is relatively easy to obtain highly intense static magnetic fields over very small volumes. Thus the emerging field of silicon micromechanics[19] (miniature motors, valves, pumps, actuators etc on a chip), and other areas of nanotechnology[20] may find use of this effect. Dielectric materials that do not exhibit piezoelectric effect may provide an alternative to piezoelectric crystals in some applications. The combined dielectric and piezoelectric response of ceramic dielectrics in a magnetic field may also be of interest in this context. Liquid and gaseous magnetized 'dielectric plasmas' may be considered as interesting states of matter. For example, pure water with $\epsilon \approx 80$ would experience the same force as Barium Titanate in our experiment if the product $E_o B_o$ in Eq. (11) were increased 20-fold - a reasonable practical condition. Thus, micromotion magnetohydrodynamics in dielectric fluids is an area that may be opened for discussion.[5]

Finally, we have presented theoretical reasons why the second term in Eq.(3) is unlikely to represent a mechanical force so that there should arise a time-average unidirectional force on a dielectric when both the electric and the magnetic fields are synchronously time-varying but are 90 degrees out of phase. If this suggestion is correct, further directions in practical applicability may be explored.

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10. Note in this context that the optical experiments, reviewed by Brevik *(op. cit.)* , do not directly address the Abraham force.

11. It is of some practical consequence whether or not the last term represents a mechanical force. If this term does not exist, then a time-average unidirectional force on a dielectric can be produced in crossed electric and magnetic fields where both vary sinusoidally with time, but are 90 degrees out of phase (see e.g. De, 1980, *op. cit.)* . If the term exists, the time-average force would vanish. As an example, J. E. Cox obtained a patent for a "Dipolar Force Field Propulsion System", U. S. Patent 4,663,932 issued May 12, 1987) based on the non-existence of the last term.

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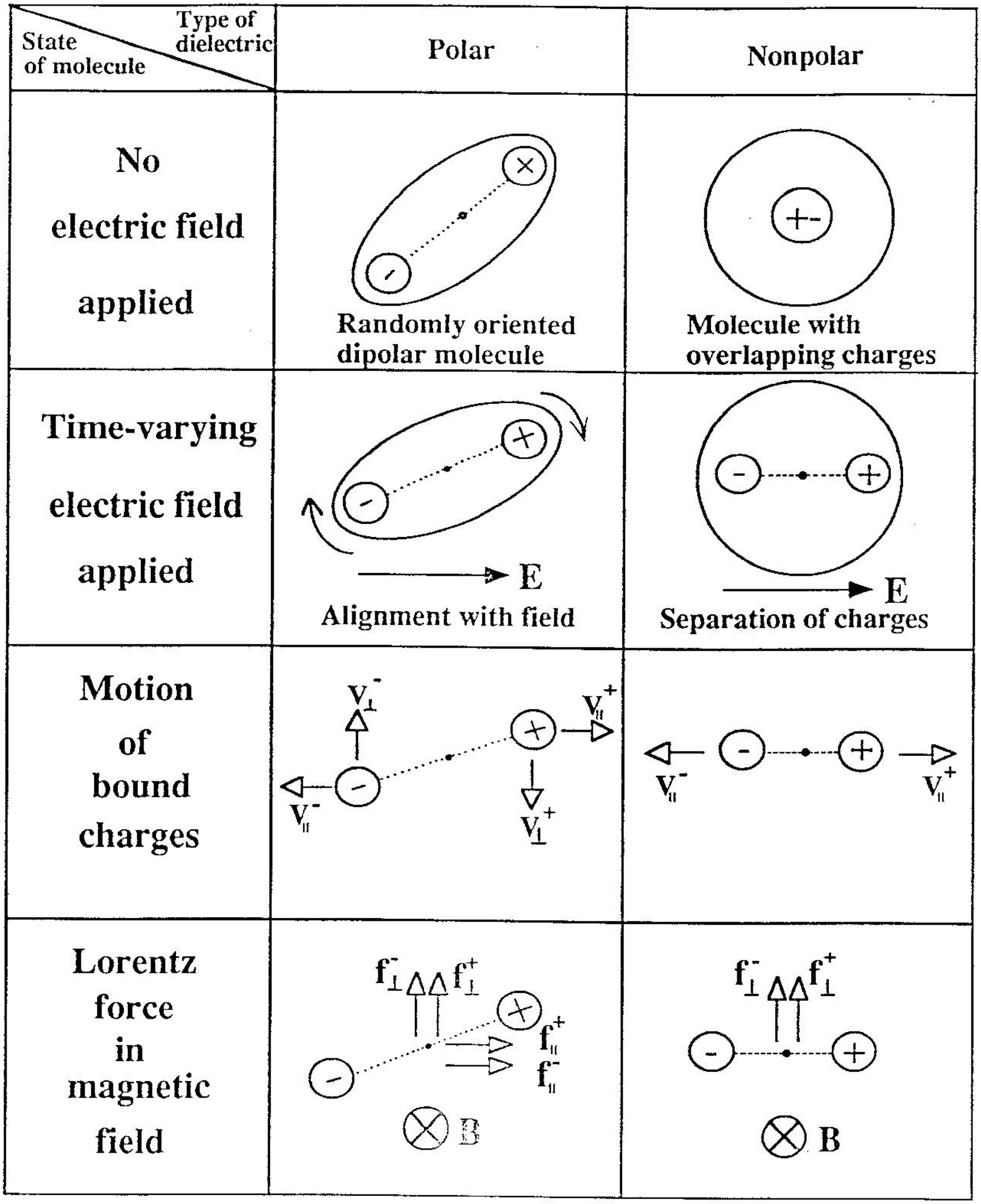
FIGURE CAPTIONS

FIG. 1 Microscopic view of the arising of a net force on a molecule of a dielectric (polar or nonpolar) carrying a polarization current perpendicular to an ambient magnetic field **B**. The magnetic field points into the plane of the paper.

FIG. 2 Experimental set-up for the observation of the mechanical force on dielectric polarization current in a magnetic field (not drawn to scale).

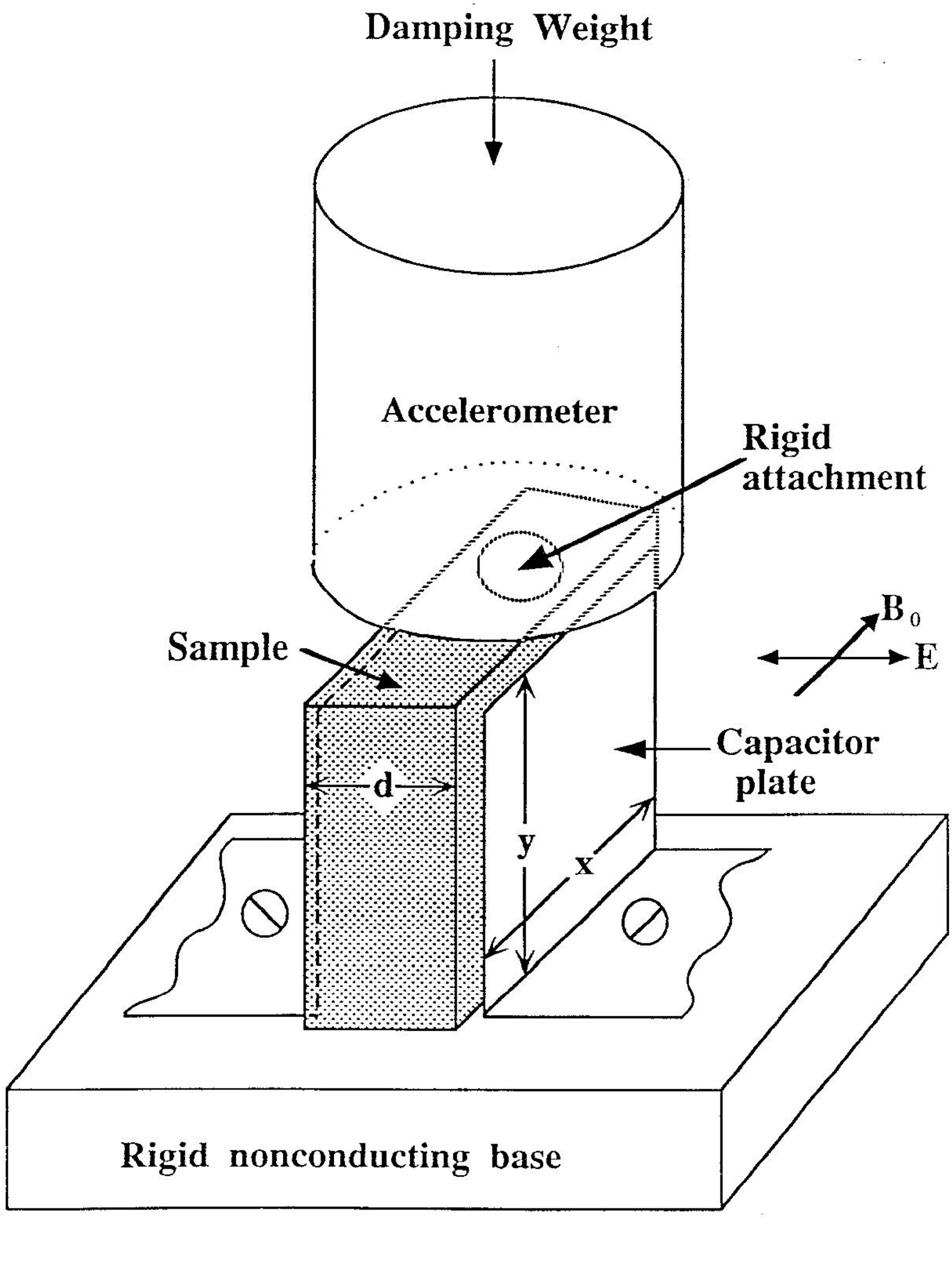
FIG. 3 (a) Detailed measurement data for Lead Zirconate Titanate ceramic dielectric at 3 KHz showing the presence of a mechanical force on the dielectric when placed in a magnetic field. The voltage V_o is due to the piezoelectric response of the dielectric. The high frequency component of the voltage is due to the resonance of the accelerometer. (b) The voltage V_p for Barium Titanate at 2.5 KHz. The dotted curves are the voltages obtained upon digitally filtering out the high frequency component.

FIG. 4 The Fourier power spectra corresponding to the last three panels of Fig. 3a. The ordinates have the same (arbitrary) unit in the three panels.



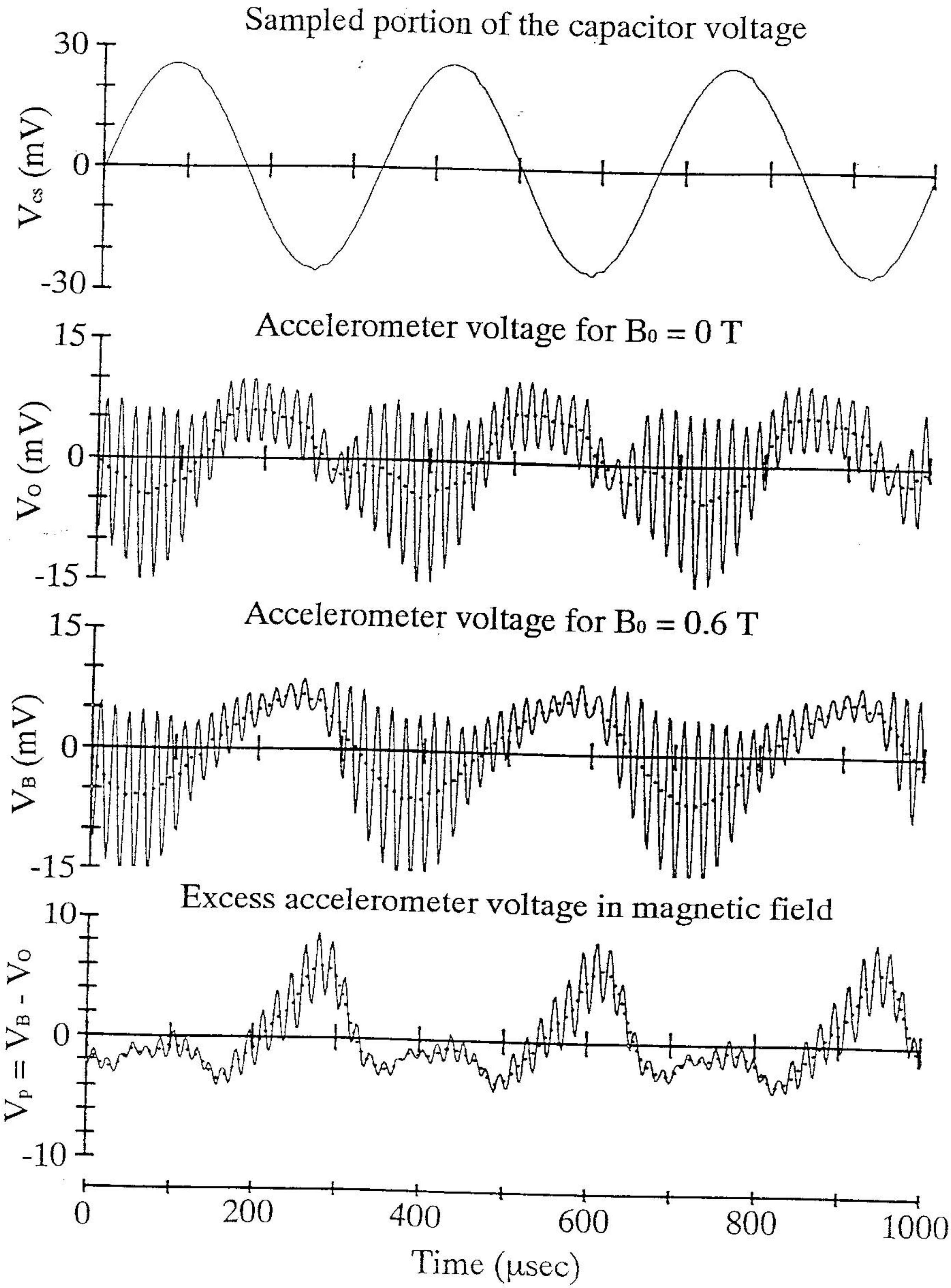
Type of dielectric
State of molecule
Polar
Nonpolar
No electric field applied
Randomly oriented dipolar molecule
Molecule with overlapping charges
Time-varying electric field applied
E
Alignment with field
E
Separation of charges
Motion of bound charges
Lorentz force in magnetic field
B
B

Fig. 1



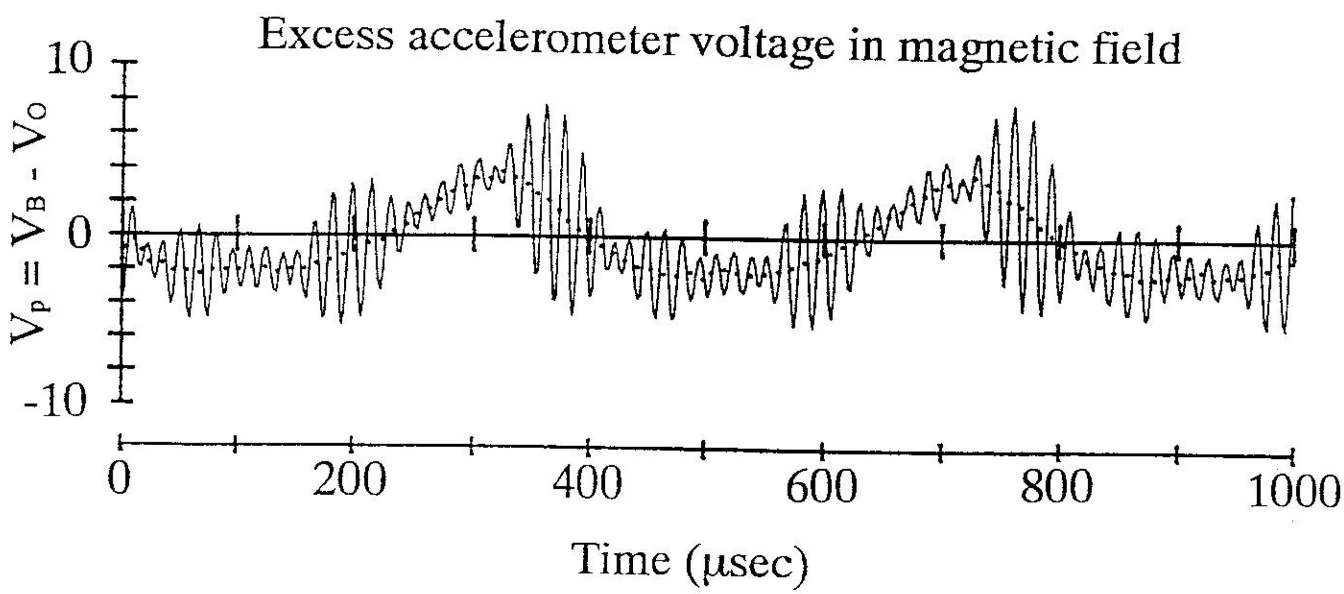
Damping Weight
Accelerometer
Rigid
attachment
B_0
E
Sample
Capacitor
plate
d
y
x
Rigid nonconducting base

Fig. 2



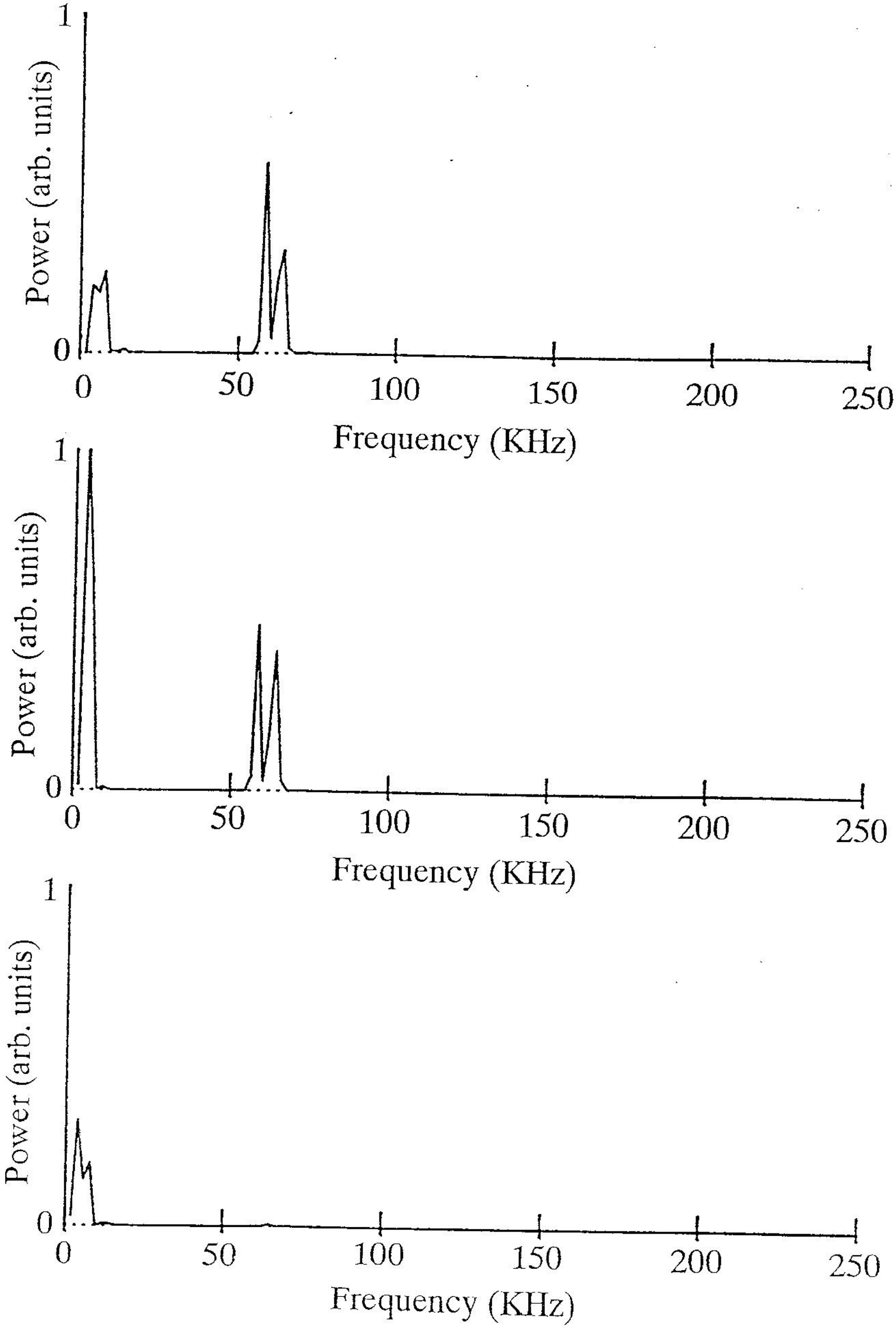
Sampled portion of the capacitor voltage
30
0
-30
V_{cs} (mV)
Accelerometer voltage for $B_0 = 0$ T
15
0
-15
V_O (mV)
Accelerometer voltage for $B_0 = 0.6$ T
15
0
-15
V_B (mV)
Excess accelerometer voltage in magnetic field
10
0
-10
$V_p = V_B - V_O$
0
200
400
600
800
1000
Time (μsec)

Fig. 3(a)



Excess accelerometer voltage in magnetic field
10
0
-10
$V_P = V_B - V_O$
0
200
400
600
800
1000
Time (μsec)

Fig. 3(b)



1
Power (arb. units)
0
0
50
100
150
200
250
Frequency (KHz)
1
Power (arb. units)
0
0
50
100
150
200
250
Frequency (KHz)
1
Power (arb. units)
0
0
50
100
150
200
250
Frequency (KHz)

Fig. 4